**Math 10A**
**Worksheet, Discussion #8; Wednesday, 6/27/2018**
**Instructor name: Roy Zhao**

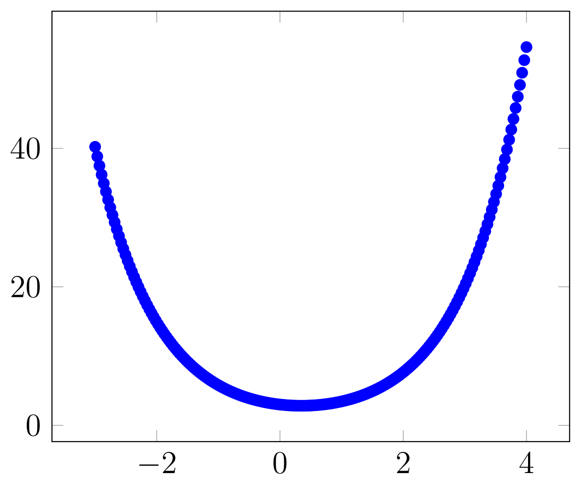
# 1 Graphing Functions

1. Sketch the graph of $f(x) = e^x + 2e^{-x}$.

**Solution:** Take the derivative and second derivative to get $f'(x) = e^x - 2e^{-x}$ and $f''(x) = e^x + 2e^{-x}$. We want to make a table and the values we care about are when $f'(x) = 0$, $f''(x) = 0$, and when they are not defined. They are always defined and solving $f'(x) = 0$ gives $e^{2x} = 2$ so $x = \frac{\ln 2}{2}$, and $f''(x) = 0$ has no solutions. So the point we need to put in is just $x = \frac{\ln 2}{2}$. We fill out the table the sign of $f', f''$ on these intervals to get

| | $(-\infty, (\ln 2)/2)$ | $(\ln 2)/2$ | $((\ln 2)/2, \infty)$ |
|---|---|---|---|
| $f'(x)$ | $-$ | 0 | $+$ |
| $f''(x)$ | $+$ | $+$ | $+$ |

Now we calculate the limits as $x \to \pm\infty$. We have $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = \infty$. We can now use this to produce something similar to the following graph noting that $f$ will have a local minimum at $x = \frac{\ln 2}{2}$ by the second derivative test.

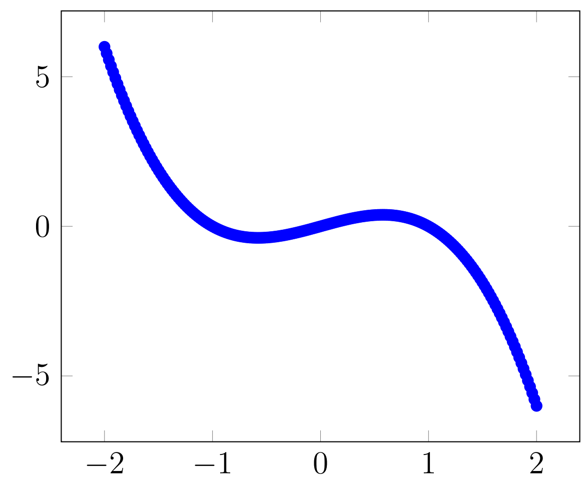



2. Sketch the graph of $f(x) = x - x^3$.

**Solution:** Take the derivative and second derivative to get $f'(x) = 1 - 3x^2$ and $f''(x) = -6x$. We want to make a table and the values we care about are when $f'(x) = 0$, $f''(x) = 0$, and when they are not defined. They are always defined and solving $f'(x) = 0$ gives $x^2 - 1/3 = 0$ so $x = \pm 1/\sqrt{3}$, and $f''(x) = 0$ gives $x = 0$. So the points we need to put in our table are $x = 0, \pm 1/\sqrt{3}$. We fill out the table the sign of $f', f''$ on these intervals to get

| | $(-\infty, -1/\sqrt{3})$ | $-1/\sqrt{3}$ | $(-1/\sqrt{3}, 0)$ | $0$ | $(0, 1/\sqrt{3})$ | $1/\sqrt{3}$ | $(1/\sqrt{3}, \infty)$ |
|---|---|---|---|---|---|---|---|
| $f'(x)$ | $-$ | $0$ | $+$ | $+$ | $+$ | $0$ | $-$ |
| $f''(x)$ | $+$ | $+$ | $+$ | $0$ | $-$ | $-$ | $-$ |

Now we calculate the limits as $x \to \pm\infty$. We have $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = \infty$. We can now use this to produce something similar to the following graph noting that $f$ will have a local minimum at $x = -1/\sqrt{3}$ and maximum at $x = 1/\sqrt{3}$ by the second derivative test.



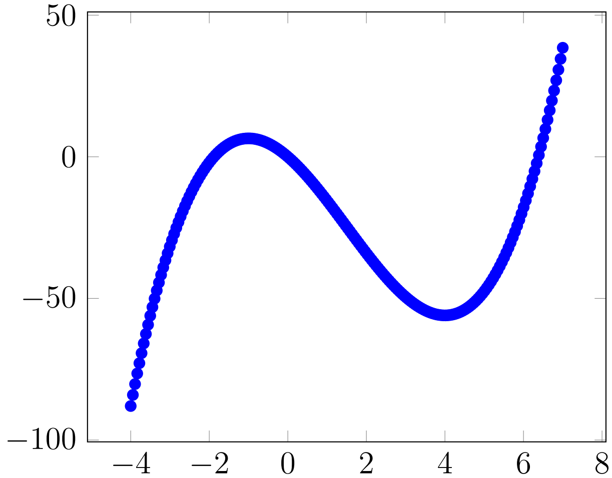

3. Sketch the graph of $f(x) = -12x - \dfrac{9x^2}{2} + x^3$.

**Solution:** Take the derivative and second derivative to get $f'(x) = -12 - 9x + 3x^2$ and $f''(x) = 6x - 9$. We want to make a table and the values we care about are when $f'(x) = 0$, $f''(x) = 0$, and when they are not defined. They are always defined and solving $f'(x) = 0$ gives $x^2 - 3x - 4 = 0$ so $x = -1, 4$, and $f''(x) = 0$ gives $x = 3/2$. So the points we need to put in our table are $x = -1, 3/2, 4$. We fill out the table the sign of $f', f''$ on these intervals to get

| | $(-\infty, -1)$ | $-1$ | $(-1, 1.5)$ | $1.5$ | $(1.5, 4)$ | $4$ | $(4, \infty)$ |
|---|---|---|---|---|---|---|---|
| $f'(x)$ | $+$ | $0$ | $-$ | $-$ | $-$ | $0$ | $+$ |
| $f''(x)$ | $-$ | $-$ | $-$ | $0$ | $+$ | $+$ | $+$ |

Now we calculate the limits as $x \to \pm\infty$. We have $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = \infty$. We can now use this to produce something similar to the following graph noting that

$f$ will have a local minimum at $x = 4$ and maximum at $x = -11$ by the second derivative test.

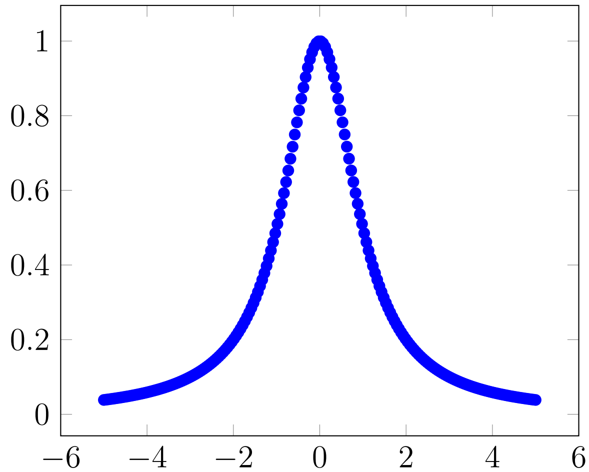



4. Sketch the graph of $f(x) = \dfrac{1}{1+x^2}$.

**Solution:** Take the derivative and second derivative to get $f'(x) = \dfrac{-2x}{(x^2+1)^2}$ and $f''(x) = \dfrac{6x^2-2}{(x^2+1)^3}$. We want to make a table and the values we care about are when $f'(x) = 0$, $f''(x) = 0$, and when they are not defined. They are always defined and solving $f'(x) = 0$ gives $x = 0$, and $f''(x) = 0$ gives $6x^2 - 2 = 0$. So the points we need to put in our table are $x = 0, \pm 1/\sqrt{3}$. We fill out the table the sign of $f', f''$ on these intervals to get

| | $(-\infty, -1/\sqrt{3})$ | $-1/\sqrt{3}$ | $(-1/\sqrt{3}, 0)$ | $0$ | $(0, 1/\sqrt{3})$ | $1/\sqrt{3}$ | $(1/\sqrt{3}, \infty)$ |
|---|---|---|---|---|---|---|---|
| $f'(x)$ | $+$ | $+$ | $+$ | $0$ | $-$ | $-$ | $-$ |
| $f''(x)$ | $+$ | $0$ | $-$ | $-$ | $-$ | $0$ | $+$ |

Now we calculate the limits as $x \to \pm\infty$. We have $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 0$. So there is a horizontal asymptote at $y = 0$. We can now use this to produce something similar to the following graph noting that $f$ will have a local maximum at $x = 0$ by the second derivative test.

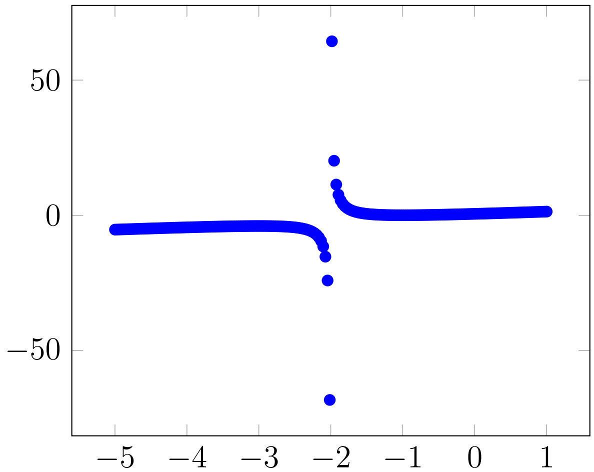



5. Sketch the graph of $f(x) = x + \dfrac{1}{2+x}$.

**Solution:** Take the derivative and second derivative to get $f'(x) = 1 - \dfrac{1}{(x+2)^2}$ and $f''(x) = \dfrac{2}{(x+2)^3}$. We want to make a table and the values we care about are when $f'(x) = 0$, $f''(x) = 0$, and when they are not defined. They are not defined when $x = -2$ and solving $f'(x) = 0$ gives $(x+2)^2 = 1$ so $x = -3, -1$, and $f''(x) = 0$ has no solutions. So the points we need to put in our table are $x = -3, -2, -1$. We fill out the table the sign of $f', f''$ on these intervals to get

| | $(-\infty, -3)$ | $-3$ | $(-3, -2)$ | $-2$ | $(-2, -1)$ | $-1$ | $(-1, \infty)$ |
|---|---|---|---|---|---|---|---|
| $f'(x)$ | $+$ | 0 | $-$ | DNE | $-$ | 0 | $+$ |
| $f''(x)$ | $-$ | $-$ | $-$ | DNE | $+$ | $+$ | $+$ |

Now we calculate the limits as $x \to \pm\infty$. We have $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = \infty$. Then since $f$ is not defined at $x = -2$, we calculate the limits of $f$ there with $\lim_{x\to-2^-} f(x) = -\infty$, $\lim_{x\to-2^+} f(x) = \infty$. So there is a vertical asymptote at $x = -2$. We can now use this to produce something similar to the following graph noting that $f$ will have a local minimum at $x = -1$ and maximum at $x = -3$ by the second derivative test.

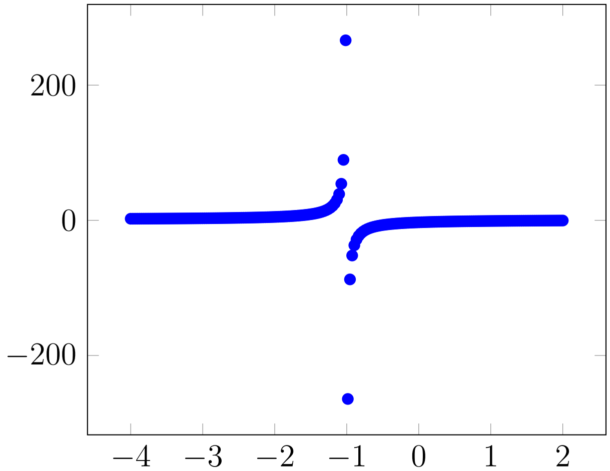



6. Sketch the graph of $f(x) = \dfrac{x-3}{x+1}$.

**Solution:** Take the derivative and second derivative to get $f'(x) = \dfrac{4}{(x+1)^2}$ and $f''(x) = \dfrac{-2}{(x+1)^2}$. We want to make a table and the values we care about are when $f'(x) = 0$, $f''(x) = 0$, and when they are not defined. They are not defined at $x = -1$ and solving $f'(x) = 0, f''(x) = 0$ has no solutions. So the points we need to put in our table is just . We fill out the table the sign of $f', f''$ on these intervals to get

| | $(-\infty, -1)$ | $-1$ | $(-1, \infty)$ |
|---|---|---|---|
| $f'(x)$ | $+$ | DNE | $+$ |
| $f''(x)$ | $+$ | DNE | $+$ |

Now we calculate the limits as $x \to \pm\infty$. We have $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 1$. So there is a horizontal asymptote at $y = 1$. Now we calculate what happens as $x \to -1$ and we have $\lim_{x\to-1^-} f(x) = \infty$, $\lim_{x\to-1^+} f(x) = -\infty$ so it has a vertical asymptote at $x = -1$. We can now use this to produce something similar to the following graph.


200
0
−200
−4
−3
−2
−1
0
1
2